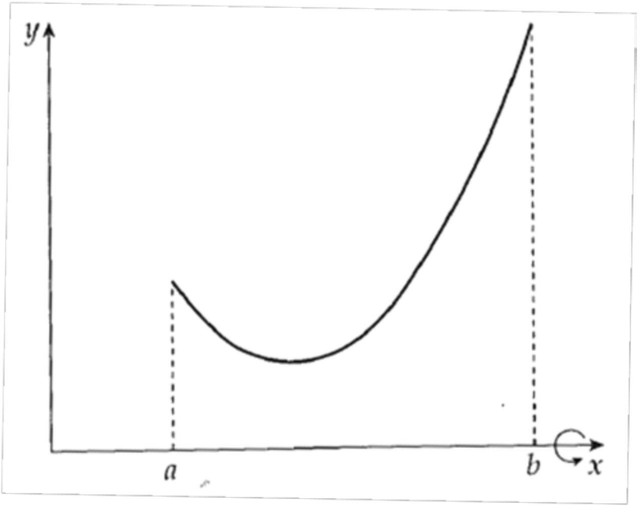
7) Volumes of revolution

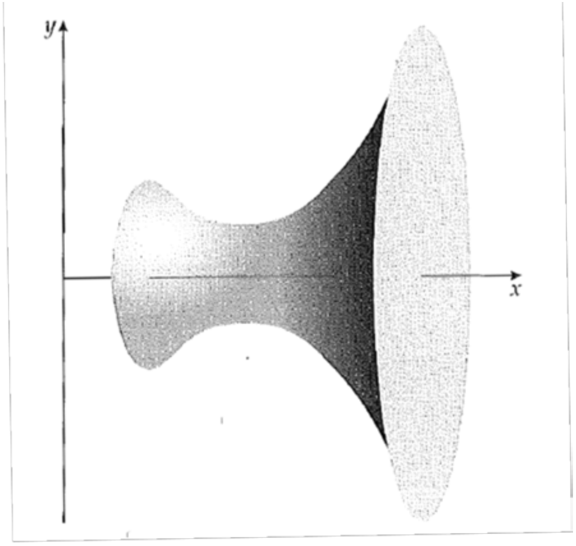
Integration can be used to find volumes of revolution. By taking an equation and rotating it about an axis, a 3-D shape can be generated, and its volume calculated. At Achieved level, only polynomial functions will be given, and drawings will be provided to help.

Demonstration:

If the function $y = f(x)$ is rotated about the x-axis on an interval $a \leq x \leq b$:



then the following three-dimensional region (below) is produced:



The formula to calculate the volume of the solid is: $\boldsymbol{V = \pi \int_a^b y^2 \; dx}$. The formula adds the volume of all the discs of very tiny width which make up the whole shape.

Example: Let the function of the curve be $y = (x-2)^2 + 1$ which can be written as $y = x^2 - 4x + 5$. Let the values of a and b be 1 and 4 respectively. Work out the volume of the solid of revolution.

$$V = \pi \int_1^4 y^2 \; dx$$

$$= \pi \int_1^4 (x^2 - 4x + 5)^2 \; dx$$

$$= \pi \int_1^4 x^4 - 8x^3 + 26x^2 - 40x + 25 \; dx$$

$$= \pi \left[\frac{x^5}{5} - 8\frac{x^4}{4} + 26\frac{x^3}{3} - 40\frac{x^2}{2} + 25x \right]_1^4$$

$$= 15.6\,\pi \text{ units}^3$$

Worksheet:
Ex A8 – Volumes of revolution
Ex A7 – Areas under curves
Ex A1 – Integrating polynomials
Ex A2 – Integrating exponentials
Ex A3 – Integrating trig functions
Ex A4 – Integrating rational functions